

# Search Complexities for HTN Planning

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# Hierarchical Task Network (HTN) planning

HTN planning is the problem of decomposing an initial task to accomplish into a sequence of executable steps:

- Often used to solve classical planning problems faster
- More expressive than classical planning
  - Can express undecidable problems

But...

- There are many ways to solve HTN problems
  - Two main algorithms in practice, two new algorithms in thesis
  - What problems are these algorithms guaranteed to solve?
  - How efficient are they?
- Search space is huge
  - Most problems in PSPACE through EXPSPACE
  - What domain independent heuristics can we design for HTN planning?
  - Can these heuristics be effective?

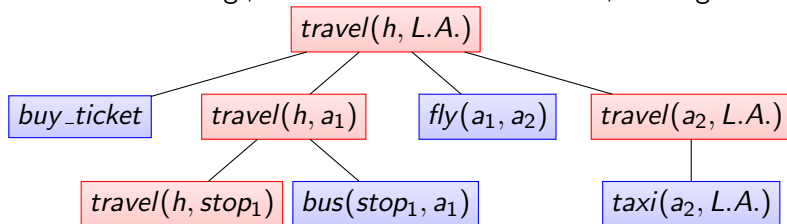
# Outline

- 1 Background
- 2 HTN Algorithms and Complexity
  - Decomposition space
  - Progression spaces
- 3 Delete-Free HTN Planning
  - Delete-Free HTN planning is NP-complete
  - Delete-Free TIHTN planning is in P
- 4 HTN Planning via translation to Classical Planning
  - Motivation
  - Translation Algorithm
  - Experiments
- 5 Conclusions

## HTN Planning (overview)

The purpose of HTN planning is to complete a *task*. Tasks are either

- *Primitive*, which corresponds to some concrete action we know how to perform
  - E.g: *walk(room, hall)*, or *drink(coffee)*
- *Non-primitive*, which is an abstract task. E.g. *travel(home, L.A.)*
- Must recursively decompose non-primitive tasks until we get primitive tasks we know how to execute directly
- Given a set of *methods*, which are recipes on how to accomplish abstract tasks. E.g., to travel from *home* to *L.A.*, we might



# Planning: Propositional and FOL

- There are two common ways to define HTN classical planning:
  - Over a propositions, and propositional sentences
  - Over a function-free fragment of first-order logic
- My work and this talk are concerned with propositional planning
  - Applies directly to non-propositional planning
  - Complexity results usually go up by an exponential amount

# HTN Planning: $D = (\mathbf{O}, M)$ , $P = (D, \mathbf{s}_0, tn_0)$

Every HTN problem  $P$  contains an *initial state*  $\mathbf{s}_0$ , which is just a collection of literals, e.g.,  $\{thirsty, tired\}$ .

*Actions* in our domain work over these states.

Actions have:

- A name (the task)
- A precondition (logical formula over the state)
  - Condition must hold for the action to be executed
- An effect on the state
  - A set of literals to add
  - A set of literals to delete
- Denote execution by a partial function  $\gamma$ :
  - $\gamma(\{thirsty, tired\}, drink) = \{tired\}$

Operator: *sleep*

pre =  $tired \wedge \neg thirsty$

eff =  $\{\neg tired\}$

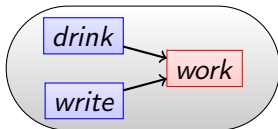
Operator: *drink*

pre = *true*

eff =  $\{\neg thirsty\}$

# HTN Planning: $D = (O, \mathbf{M})$ , $P = (D, s_0, \mathbf{tn}_0)$

- An initial *task network*,  $tn_0$ , which is a labeled DAG of *tasks* to accomplish:
  - Edges represent ordering constraints
  - Nodes are labeled with *tasks*, which are either:
    - Primitive tasks (i.e., action names from  $O$ :  $\{drink, write\}$ )
    - Non-primitive tasks (say,  $\{work\}$ )



- $M$ , a set of methods to *decompose* non-primitive tasks

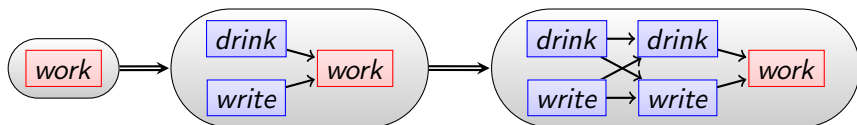
# Methods and Decomposition

- A method  $(t, tn)$  is a non-primitive task  $t$  paired with a network  $tn$

Method:



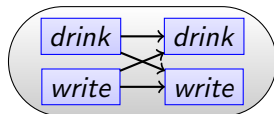
- We *decompose* a task network by replacing a node in the network with a corresponding method's network.





## Primitive task networks:

- Two states:
  - $thirsty, \neg thirsty$
- Two actions:
  - $\forall_s \gamma(s, write) = thirsty$
  - $\forall_s \gamma(s, drink) = \neg thirsty$



- All names are actions  $\Rightarrow$  network is *primitive*
- It is *executable* in a state if there exists a consistent total order (sequence) which is executable.
  - Four possible orderings
  - All executable in both  $thirsty$  and  $\neg thirsty$
  - Could end in either state.
- Objective: Decompose initial network to a primitive task network that is executable in  $s_0$ .

# Classical (STRIPS) planning

Differences between HTN and Classical Planning:

- $P = (O, s_0, G)$
- No methods and no initial task network.
- Objective: Find a plan (any executable sequence) that take us to a goal state.
  - No restriction (other than executability) on when planner can insert an action
- PSPACE-complete for propositional problems

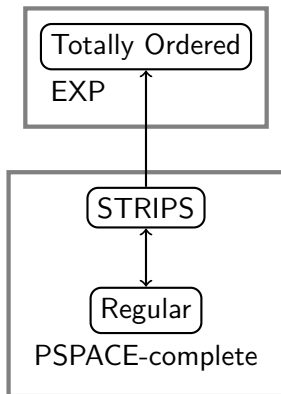
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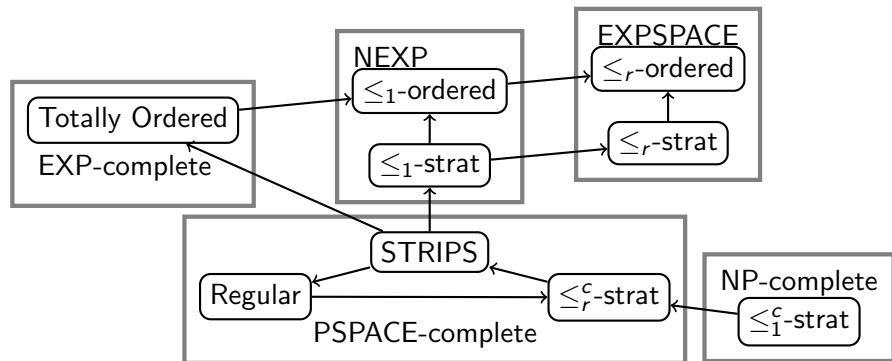
# HTN Algorithms and Complexity

- Most HTN planners are sound and complete
  - HTN planning is undecidable
  - There are problems on which any set of planners will never terminate
  - In practice, we observe differences in how well they work. Why?
- General strategy:
  - Identify what problems each HTN algorithm halts on
  - Bound the run-time of the planner on those problems
- Given these problem classes, we can propose new HTN algorithms
  - Compare them to current ones on more than just an experimental basis
  - Two new algorithms in thesis

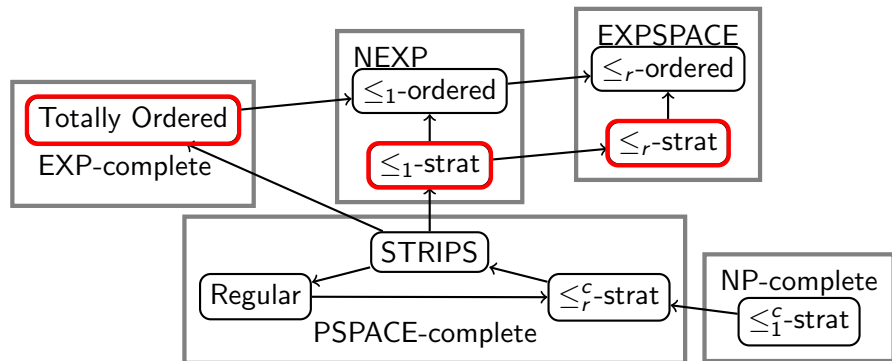
## Before thesis



## In the thesis

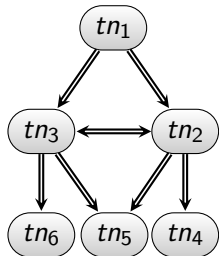


## In the thesis



# Decomposition as a graph

- UMCP (Erol 94), Landmark-aware HTN planner (Elkawkagy 2012) directly use the definition of decomposition to find solutions
- Starting at the initial task network, the choices form a graph:
  - Each vertex is a task network reachable via decomposition
  - Each edge is a decomposition
- Call this the *decomposition space* of a problem
- Plan by searching graph (not always finite)





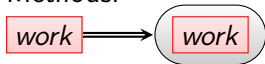
## Decomposition-decided problems: $\leq_1$ -stratification

- A domain is  $\leq_1$ -stratifiable if we can stratify its task names such that for every method  $(t, tn) \in M$ :
  - If  $tn$  has only one task, then  $tn$ 's task is on an equal or lower stratum than  $t$  (trivially recursive decomposition)
  - Otherwise, every task in  $tn$  is on a lower stratum than  $t$  (acyclic decomposition)

Constraints:

- $strat(work) \leq strat(work)$

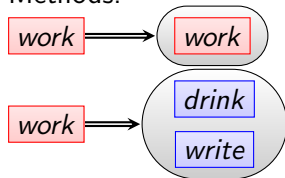
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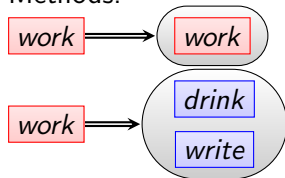
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$strat(work) = 2$
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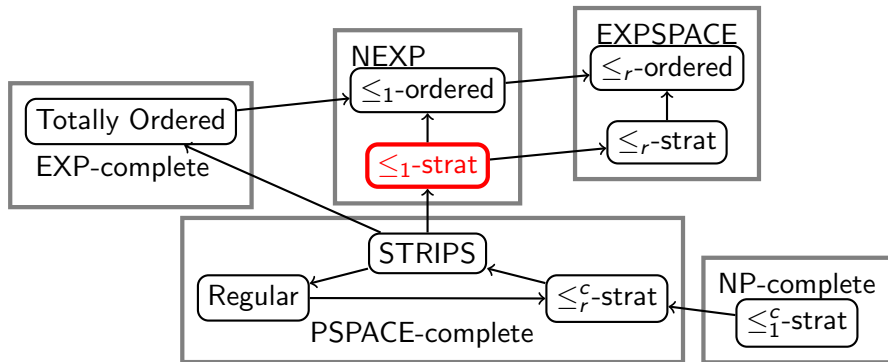
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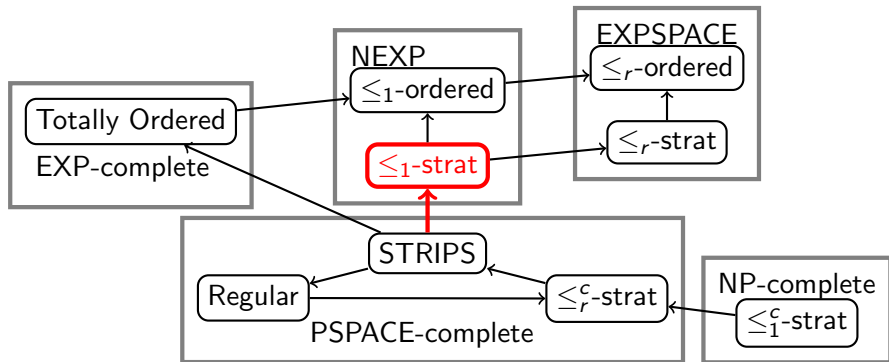
No stratification!

Decomposition spaces and  $\leq_1$ -stratifiable problems

From the thesis (but not in this talk):

- Theorem: Decomposition decides  $\leq_1$ -stratifiable problems and nothing else
- Plan existence is in NEXP for  $\leq_1$ -stratifiable problems
  - Can preprocess  $\leq_1$ -stratifiable problems into acyclic problems.

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- Theorem: Decomposition decides  $\leq_1$ -stratifiable problems and nothing else
- Plan existence is in NEXP for  $\leq_1$ -stratifiable problems
  - Can preprocess  $\leq_1$ -stratifiable problems into acyclic problems.
- But how hard is  $\leq_1$ -stratifiable planning?

# $\leq_1$ -stratifiable HTN planning is PSPACE-hard

## Propositional classical planning (Propositional STRIPS)

- Similar to HTN planning, but no methods
- Use any sequence of operators to obtain a given goal
- Plan-existence is PSPACE-complete (Erol et. al.)
- Shortest solution is always  $< 2^{|L|}$ , where  $L$  is the set of propositions in the language.

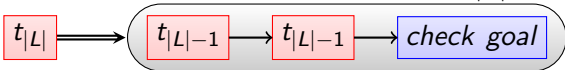
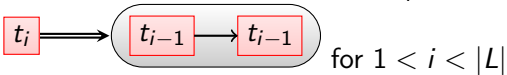
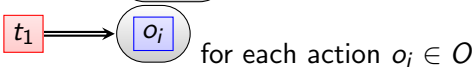
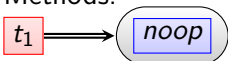


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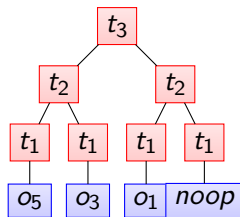
Encoding STRIPS in  $\leq_1$ -strat HTNs

Given a propositional STRIPS problem  $(L, O, s, goal)$ , create an HTN domain  $D$  with  $O, L$ , the tasks  $t_1, \dots, t_{|L|}$ , and the following methods:

Methods:



Translated HTN Problem:  $(D, s_0, t_{|L|})$



## Progression spaces for HTN planning

A number of HTN planners (SHOP, SHOP2, etc) plan for tasks in the order that they are to be executed. This is called *progression*.

For an HTN problem  $P = (D, s_0, tn_0)$ , we *progress*  $P$  by picking a task  $t$  in  $tn_0$  with no predecessors. Then:

- If  $t$  is non-primitive:
  - Decompose  $t$  in  $tn_0$ , let  $tn'$  be the result
  - Then  $(D, s_0, tn')$  is a progression of  $P$ .
- If  $t$  is primitive ( $t \in O$ ) and executable, then:
  - Apply  $t$ :  $\gamma(s_0, t) = s_1$
  - Remove it from  $tn_0$ :  $tn' = tn_0 \setminus \{t\}$
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work

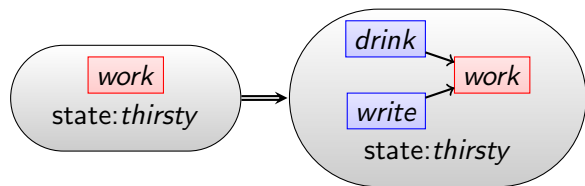
state: *thirsty*

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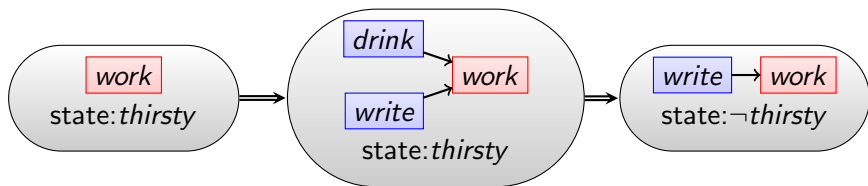


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## Progression-decided problems: $\leq_r$ -stratification

- A  $\leq_r$ -stratification of task names: For every method  $(t, tn)$ ,
  - **Last task** on equal or lower stratum (tail recursion).
  - Every other task of  $tn$  must be on lower strata.

Methods:



Constraints:

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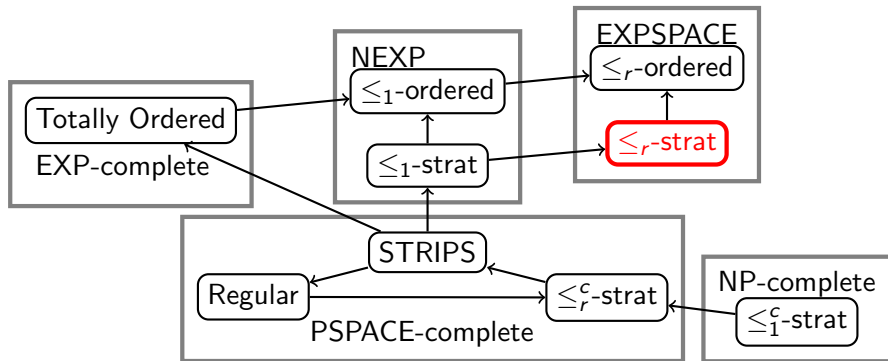
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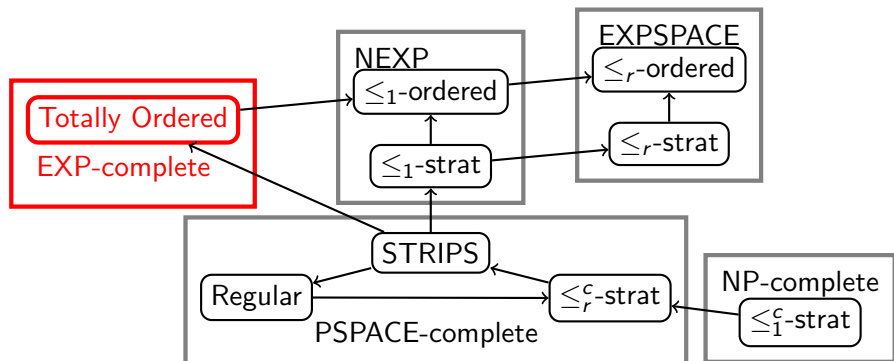
# Progression spaces and $\leq_r$ -stratifiable problems



From the thesis (but not in this talk):

- Property: Every  $\leq_1$ -stratifiable problem is  $\leq_r$ -stratifiable
- Theorems: Progression decides  $\leq_r$ -stratifiable domains
  - No larger set of progression-decided problems identifiable from methods only
  - Plan-existence in EXPSPACE for  $\leq_r$ -stratifiable since it's unclear what the bound is for length of a solution

# Totally-Ordered Planning is EXP-time complete



- Totally-Ordered HTN Problems: Initial task network and all method networks are totally ordered.
- Known to be in EXP-time (Erol et. al., TOPHTN from thesis)

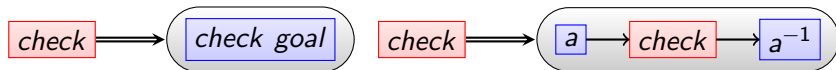


# Totally-Ordered Planning is EXP-time complete

Totally Ordered Planning:

- Good at Palindromes! The following methods always return to the same state the task 'check' starts in:

Methods:

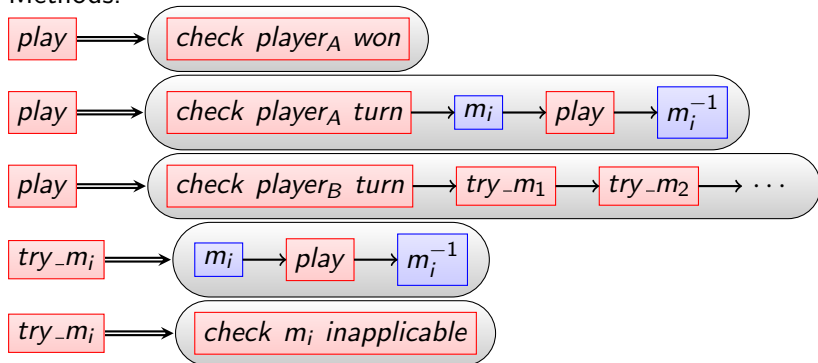


- Generalized checkers, chess, versions of Go are all EXP-complete
- We can use palindrome like methods to encode position evaluation as an HTN problem

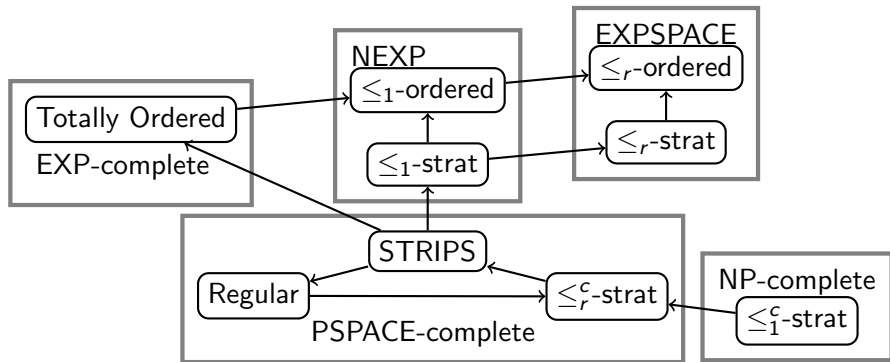
# Encoding game tree search

Encoding game tree search: Can  $player_A$  force a win?

Methods:



# Summary



Characterized four different problem spaces for HTN problems:

- Provided terminating HTN planning algorithms for each space.
- Provided syntactic checks for finiteness for each space.
- Provided expressivity and running-times for syntactically identifiable problems

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# Heuristics

Domain independent heuristics in search:

- Problem spaces are still intractably large to search
- A heuristic underestimates the distance to nearest solution, or we pretend it does
- Usually based on solving a *relaxed problem*
  - Should take low-order polynomial time to solve
  - Solution to full problem implies solution to relaxed problem (but not vice versa)

# Delete-Free Planning

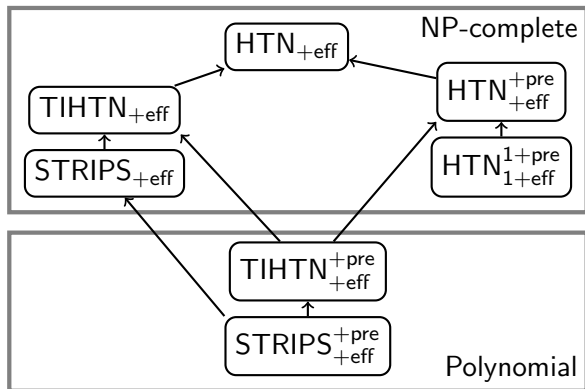
## Delete-Free Planning In STRIPS:

- All operators have positive preconditions and effects; positive goal
- Poly-time to find a plan: just apply operators until a fixed point is reached
- NP-hard to find optimal plan
- Basis for the most influential classical planning heuristics

## Delete-Free HTN Planning:

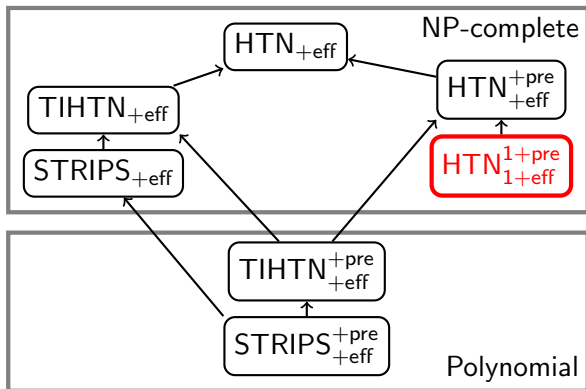
- Positive preconditions and effects
- Decidability and complexity previously unknown

# Delete-free HTN planning is NP-complete



- Complexity of plan-existence for propositional delete-free STRIPS and HTN planning with various restrictions. Arrows represent subclass relationships
- The STRIPS results are from Bylander; the other results are new

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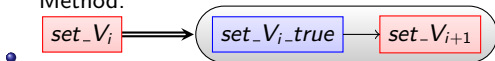
# Delete-Free HTN planning is NP-hard

CNF-SAT:  $(V_1 \vee \neg V_3 \vee V_4) \wedge (V_2 \vee V_3 \vee \neg V_4) \wedge \dots$

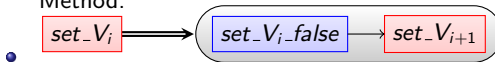
Translation of setting variables to delete-free HTN planning:

- For each variable  $V_i$ :
  - Two propositions:  $V_i\_true$ ,  $V_i\_false$
  - Two test operators:  $check\_V_i\_true$ ,  $check\_V_i\_false$
  - Two set operators:  $set\_V_i\_true$ ,  $set\_V_i\_false$
  - Two methods:

Method:



Method:



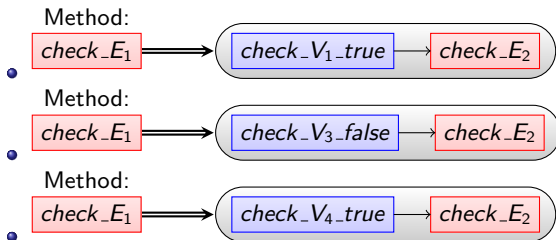
- For last variable, replace last task with  $check\_E_1$ , instead

# Delete-Free HTN planning is NP-hard

CNF-SAT:  $(V_1 \vee \neg V_3 \vee V_4) \wedge (V_2 \vee V_3 \vee \neg V_4) \wedge \dots$

Translation of testing the clauses to delete-free HTN planning:

- For disjunctive clause  $E_i$ :
  - A method for each literal



- For last clause, drop the final *check* task

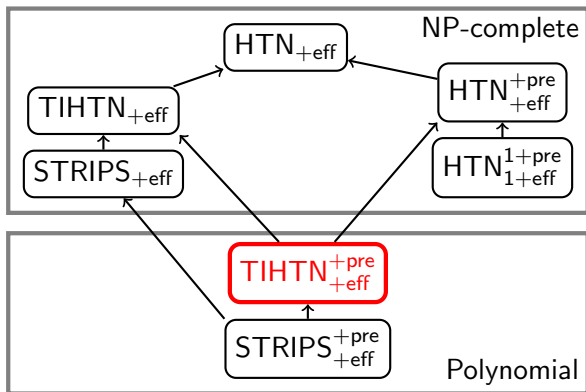
# Delete-Free HTN planning is NP-complete

- Polynomial encoding of CNF-SAT means delete-free HTN planning is NP-hard.
- We can also show delete-free HTN planning is in NP(in thesis), so it is NP-complete.
- So we need to relax the semantics of HTN planning to find a poly-time heuristic.

# Delete-Free Task-Insertion HTN Planning

- Need to relax HTN semantics to find poly-time heuristic
- Task-Insertion HTN (TIHTN) Planning:
  - Allows insertion of arbitrary operators into task network
- Delete-free TIHTN planning is poly-time:
  - Insert operators until you reach a fixpoint
  - Test to see if tasks are executable in the fixpoint state (similar to testing if a CFG is empty).

## Relaxed-semantics HTN heuristics exist



- So there exist poly-time heuristics for HTN planning. But if they deviate significantly from HTN semantics, will they still be useful?

# Outline

- 1 Background
- 2 HTN Algorithms and Complexity
  - Decomposition space
  - Progression spaces
- 3 Delete-Free HTN Planning
  - Delete-Free HTN planning is NP-complete
  - Delete-Free TIHTN planning is in P
- 4 HTN Planning via translation to Classical Planning
  - Motivation
  - Translation Algorithm
  - Experiments
- 5 Conclusions

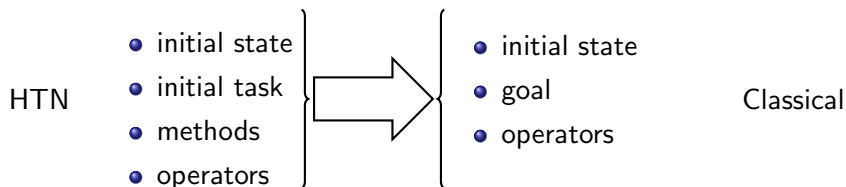
# Translating to Classical Planning:

## Motivation

- Variety of domain independent heuristics
- Highly tuned search implementations
- Want to make use of these for HTN planning!

# HTN Planning via translation to Classical Planning

- Both STRIPS and totally-ordered  $\leq_r$ -stratifiable planning are PSPACE-complete
- We can translate any totally-ordered  $\leq_r$ -stratifiable HTN problem into a classical planning problem.
- To do this, we translate the progression space of an HTN planner into the state space of a classical planner:



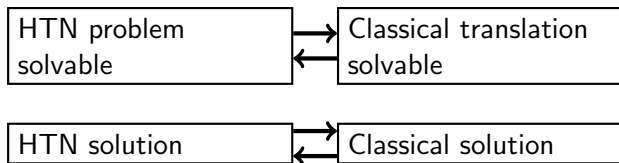
- (detail in the thesis)



# Formal Properties

Given an  $\leq_r$ -stratifiable problem:

- A solution to the translated problem  $\iff$  a solution to the HTN problem
- Any classical planner is now an HTN planner (free heuristic function to guide the search and often guaranteed termination).



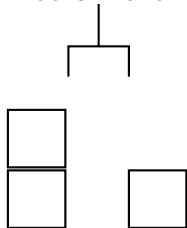
# Experiments

- Took FastForward (FF) [Hoffmann and Nebel, 2001]
  - FF is a well known classical planner
  - Investigated how well FF did on its own
  - Investigated how FF did with a translated HTN
- Compared against SHOP2 [Nau et al., 2003]
  - SHOP2 is an influential HTN planner
  - Used here with depth-first search
- 3 domains, about 5000 experiments:
  - Towers of Hanoi
  - Blocks World
  - An office delivery domain

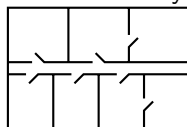
### Towers of Hanoi



### Blocks World

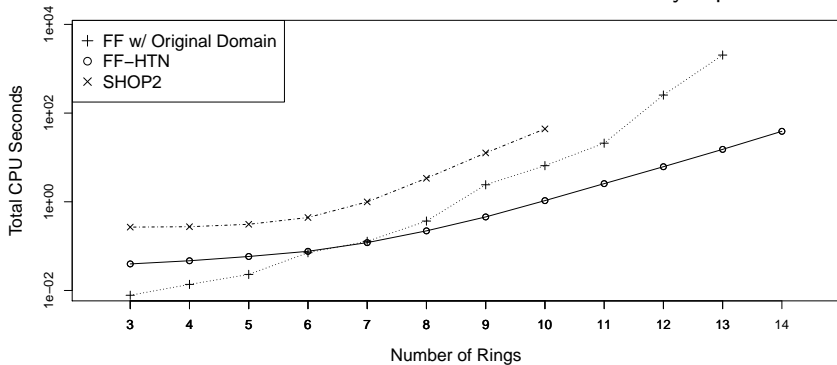


### Office Delivery



# Results: Towers of Hanoi

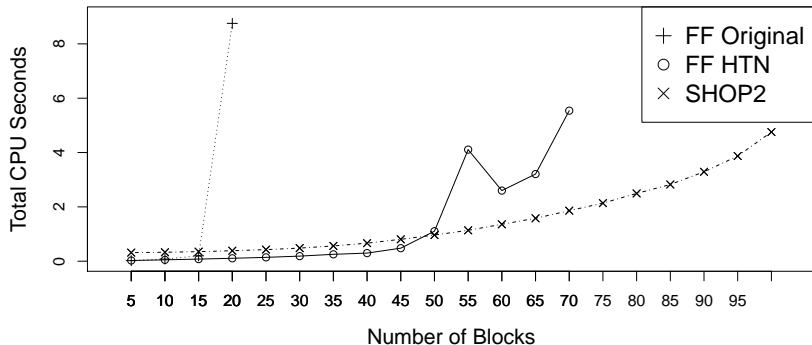
- Exponentially long plans
- No branching in the HTN domain
- Average of 100 runs / point
- FF with the translated HTNs runs exponentially faster
- SHOP2 timed out after 10 rings
  - Most likely a performance bug



# Results: Blocks World Time

## HTN description

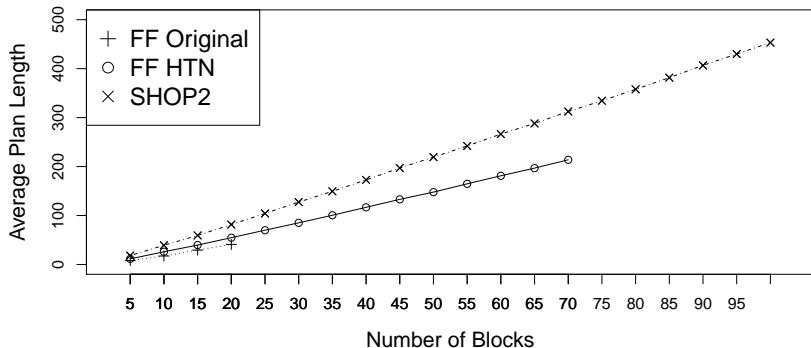
- Forces a polynomial length plan
- Does not enforce optimal solution
- FF's heuristic is known poor at blocks world
- FF with translated HTNs can solve problems up to 70 blocks in size
- SHOP2 scaled better after 50 blocks.



# Results: Blocks World Plan Length

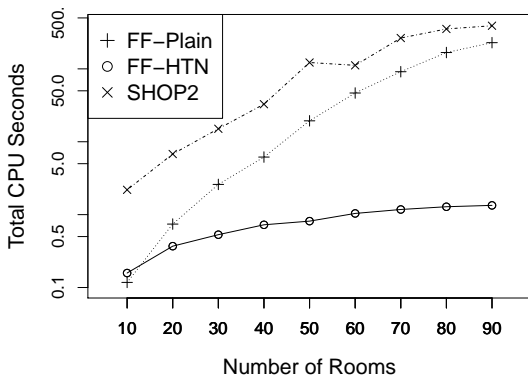
## HTN description

- Forces a polynomial length plan
- Does not enforce optimal solution
- FF with HTNs produced better plans than SHOP2
- HTNs forbid some optimal solutions, hence the plans were not the shortest possible



## Results: An Office Delivery Domain

- Office delivery: Pick up packages, deliver to another room.
- HTNs orders 'pickup'/'putdown' actions
- No guidance on what room to move to.



- SHOP2 has no heuristic to guide navigation
- FF without HTNs is poor at package management
- FF with HTNs returns plans exponentially faster than either of the above.

# Summary

- Four problems spaces for HTN planning
  - Syntactic checks for when those spaces must be finite
  - Algorithms to search these spaces (unimplemented)
  - Expressivity and running times for identified spaces
- Theoretical results impacting HTN heuristics:
  - Delete-free HTN planning is NP-complete
  - Poly-time relaxed-semantics variants exist
- An HTN-to-Classical translation for any totally-ordered  $\leq_r$ -stratifiable problem

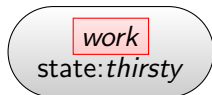
# Contributions

Provided:

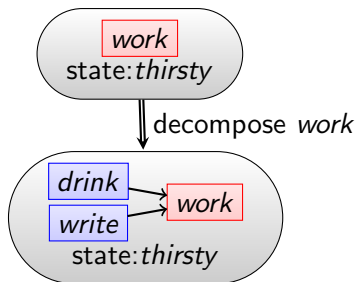
- Theoretical background to compare HTN planning search algorithms
- Theoretical limits on fidelity of HTN heuristics
- Evidence that heuristic search is effective for HTN planning



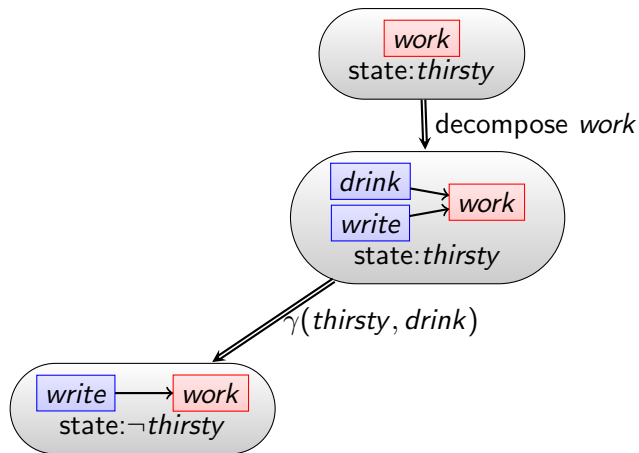
## Progression Example (over problems, not task networks)



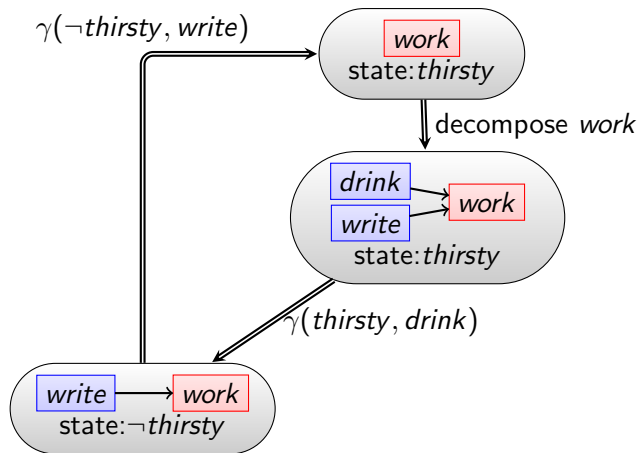
## Progression Example (over problems, not task networks)



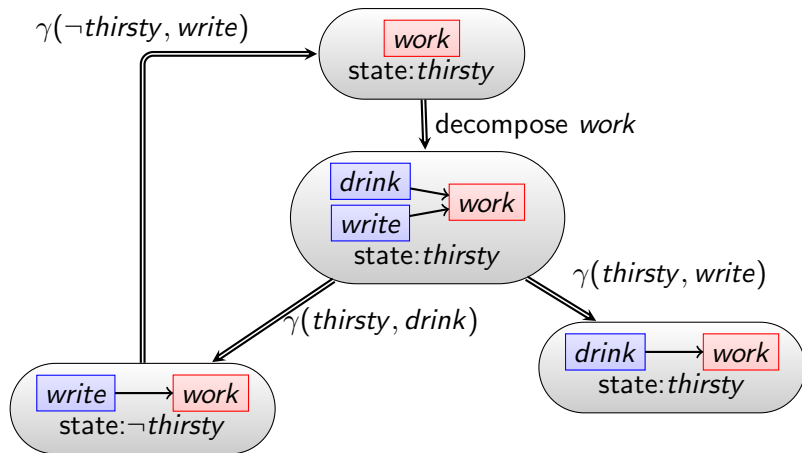
# Progression Example (over problems, not task networks)



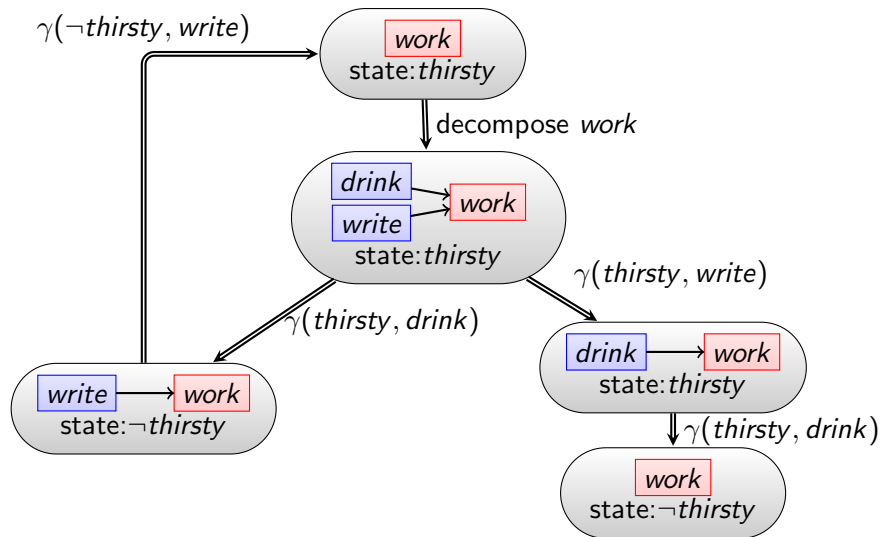
# Progression Example (over problems, not task networks)



# Progression Example (over problems, not task networks)



# Progression Example (over problems, not task networks)



## Totally-Ordered $\leq_r$ -stratifiable problems

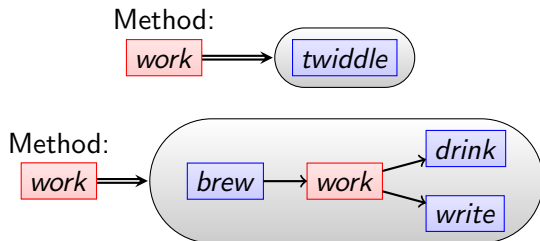
- Totally-Ordered HTN Problems: Initial task network and all method networks are totally ordered.
- Theorem: Totally-ordered  $\leq_r$ -stratifiable planning is PSPACE-complete. Proof:
  - Hardness: Contains *regular* HTN planning (Erol et. al.)
  - Progression space contains only totally-ordered problems
  - Progression can only add lower stratum tasks to the beginning of the task network
  - So  $\langle 4, 4, 5, 5 \rangle \rightarrow \langle 3, 3, 3, 4, 4, 5, 5 \rangle$
  - Max network size:  $m \cdot h$ , where  $m$  size of largest method,  $h$  height of stratification.
- Contains totally-ordered  $\leq_1$ -stratifiable planning. Searching progression space is optimal, searching decomposition space is not.

# Motivating Example: Morning coffee

- Brew as much coffee in the morning as you need throughout the day.
- Not  $\leq_r$ -stratifiable ('work' now in the middle)
  - 'work' is a subtask of itself, but not the last task.

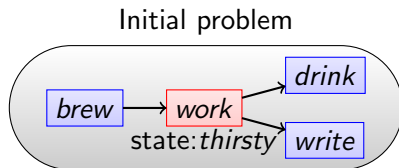
Domain actions:

- $\forall_s \gamma(s, \text{brew}) = s$
- $\forall_s \gamma(s, \text{twiddle}) = s$
- $\forall_s \gamma(s, \text{write}) = \text{thirsty}$
- $\forall_s \gamma(s, \text{drink}) = \neg \text{thirsty}$





## Total Order Partition spaces



- Plan by splitting problems into smaller problems
- If there is a total order over chunks of the task network:
  - Serialize network into smaller chunks
  - End state of one is the start state of the next.
  - If last chunk solvable, then the whole problem is solvable
- When no total order, use progression
- Call this the Total Order Progression (TOP) space
  - No existing planner terminates on all problems with a finite TOP space
  - Possible to write an algorithm that does (see thesis for details)

# Total Order Partition spaces

$\leq_r$ -ordered

- For finiteness, need to consider the structure of networks in the problem.
- A task network  $tn$  is  $\leq_r$ -ordered if  $\langle tn_1, \dots, tn_n \rangle$  are the totally ordered chunks of  $tn$  and every  $tn_i$  is either:
  - singular (only one task)
  - $\leq_r$ -stratifiable
- A problem is  $\leq_r$ -ordered if the initial task network and all methods' task networks are  $\leq_r$ -ordered
- $\leq_1$ -ordered is defined similarly over  $\leq_1$ -stratification

# TOP spaces and $\leq_r$ -ordered problems

From the thesis (but not in this talk):

- Property: If a problem is  $\leq_r$ -stratifiable, it is  $\leq_r$ -ordered
- Property: If a problem is totally ordered, it is  $\leq_r$ -ordered
- Theorem: If a problem is  $\leq_r$ -ordered, its TOP space is finite.
  - SHOP and SHOP2 may not terminate on these problems.
  - Possible to implement a planner that does (TOPHTN).
- Plan-existence in EXPSPACE for  $\leq_r$  ordered, in NEXP for  $\leq_1$ -ordered

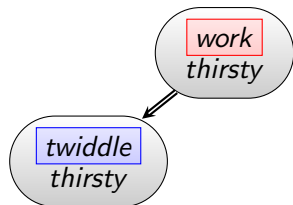
## Example



End states of  $(D, thirsty, work)$ :

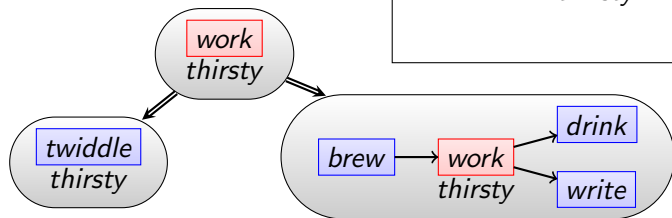
## Example

End states of  $(D, thirsty, work)$ :  
*thirsty*



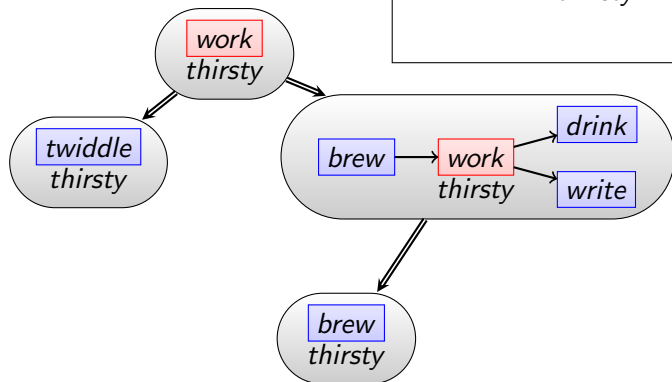
## Example

End states of  $(D, thirsty, work)$ :  
*thirsty*



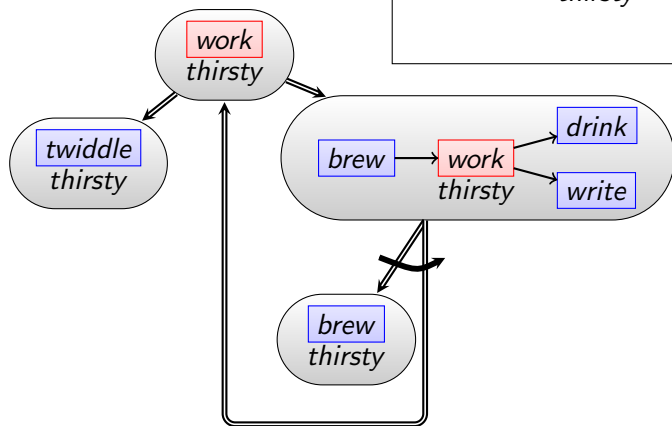
## Example

End states of  $(D, thirsty, work)$ :  
*thirsty*



## Example

End states of  $(D, thirsty, work)$ :  
*thirsty*



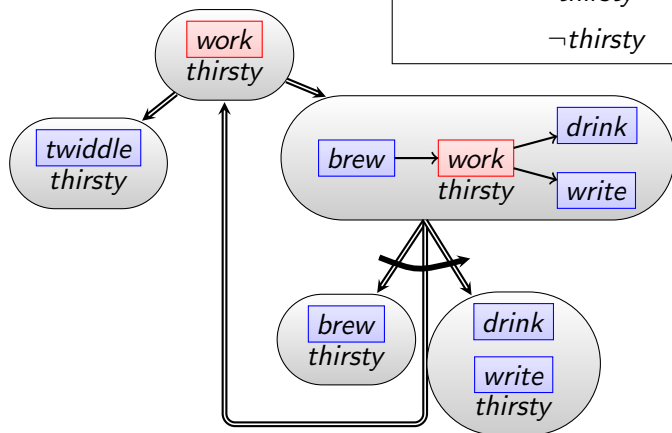


# Example

End states of  $(D, thirsty, work)$ :

$thirsty$

$\neg thirsty$

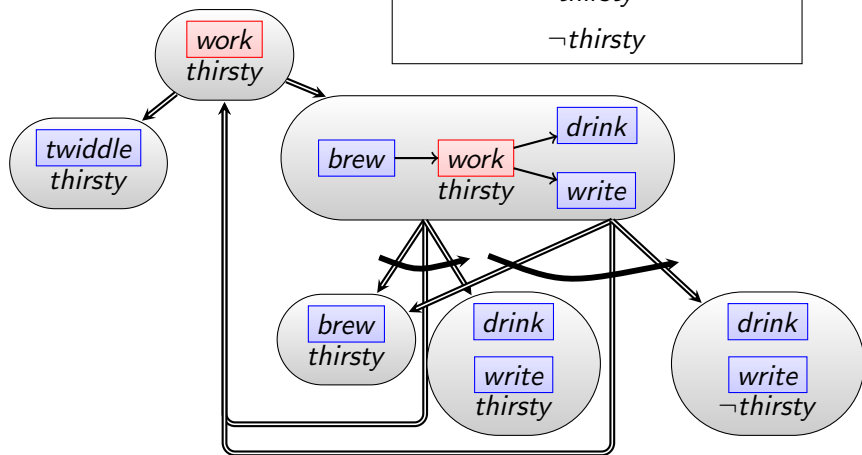


# Example

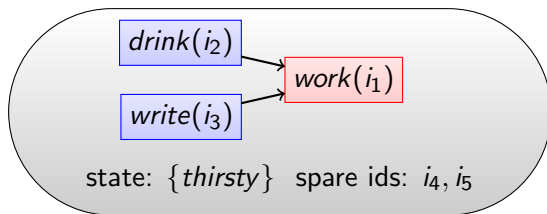
End states of  $(D, thirsty, work)$ :

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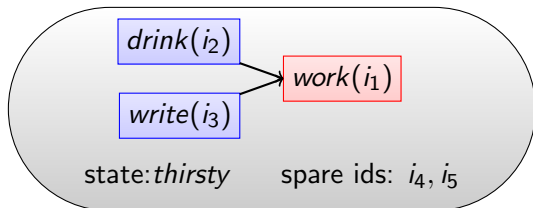


## Translating a Task Network



- Associate each task with a unique identifier:  $brew(i_1)$ ,  $drink(i_2)$ ,  $write(i_3)$ ,  $work(i_4)$
- Mark which task nodes have no preceding tasks:  $running(i_1)$
- Describe the ordering relationships between the nodes
- Add a pool of spare identifiers

# Translating Operators



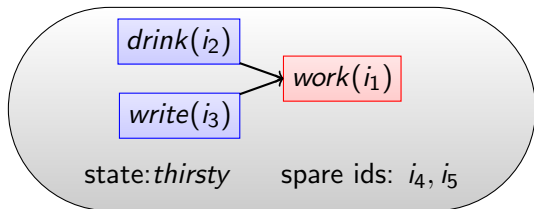
- For operators
  - Syntactically the same in HTN and classical planning
  - Semantically, can only apply an operator when it's in the task network and has no predecessors
  - Add task literal to precondition of operator

Operator:  $brew(i)$

$pre = \{ brew(i),$   
 $running(i) \dots \}$

$eff = \{ \neg brew(i),$   
 $\neg running(i),$   
 $finished(i) \dots \}$

# Translating Operators



Execute *drink*:  $\gamma(thirsty, drink(i_2)) = \neg thirsty$

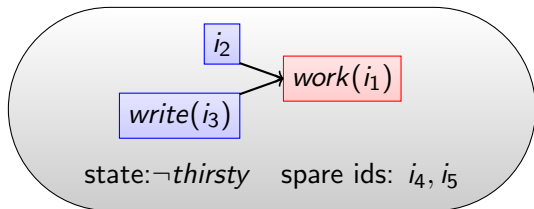
- For operators
  - Syntactically the same in HTN and classical planning
  - Semantically, can only apply an operator when it's in the task network and has no predecessors
  - Add task literal to precondition of operator

Operator: *brew*( $i$ )

pre = { *brew*( $i$ ),  
*running*( $i$ ) ... }

eff = {  $\neg$ *brew*( $i$ ),  
 $\neg$ *running*( $i$ ),  
*finished*( $i$ ) ... }

# Translating Operators



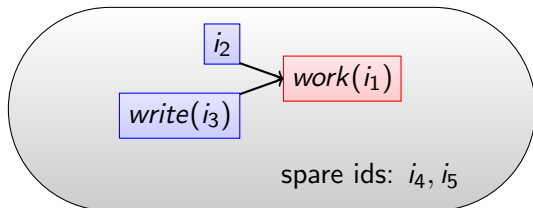
- For operators
  - Syntactically the same in HTN and classical planning
  - Semantically, can only apply an operator when it's in the task network and has no predecessors
  - Add task literal to precondition of operator

Operator:  $brew(i)$

pre = {  $brew(i),$   
 $running(i) \dots$  }

eff = {  $\neg brew(i),$   
 $\neg running(i),$   
 $finished(i) \dots$  }

# Translating Operators



Execute *write*:  $\gamma(\neg thirsty, write(i_3)) = thirsty$

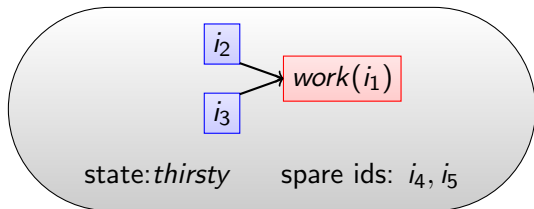
- For operators
  - Syntactically the same in HTN and classical planning
  - Semantically, can only apply an operator when it's in the task network and has no predecessors
  - Add task literal to precondition of operator

Operator:  $brew(i)$

$pre = \{ brew(i),$   
 $running(i) \dots \}$

$eff = \{ \neg brew(i),$   
 $\neg running(i),$   
 $finished(i) \dots \}$

# Translating Operators



- For operators
  - Syntactically the same in HTN and classical planning
  - Semantically, can only apply an operator when it's in the task network and has no predecessors
  - Add task literal to precondition of operator

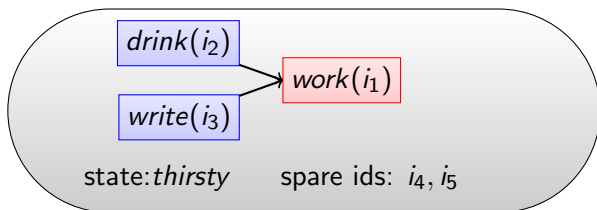
Operator:  $brew(i)$

$pre = \{ brew(i),$   
 $running(i) \dots \}$

$eff = \{ \neg brew(i),$   
 $\neg running(i),$   
 $finished(i) \dots \}$



## Managing precedence constraints



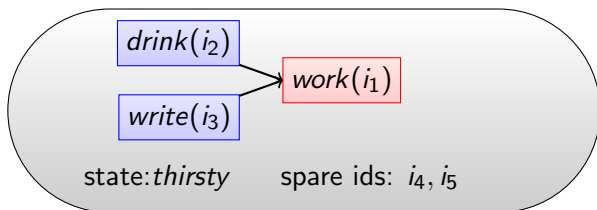
Manage precedence constraints with operators:

- Wait for tasks to be finished
- Free the identifiers
- Mark task as ready to run

Wait op:  $wait_{22}(i_1, i_2, i_3)$

$pre = \{finished(i_2), finished(i_3), \dots\}$   
 $eff = \{free\ i_2\ and\ i_3$   
 $running(i_1)\}$

## Managing precedence constraints



Execute *drink*:  $\gamma(thirsty, drink(i_2)) = \neg thirsty$

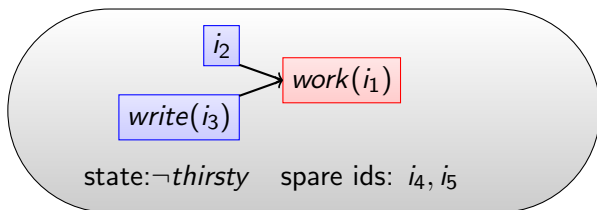
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## Managing precedence constraints



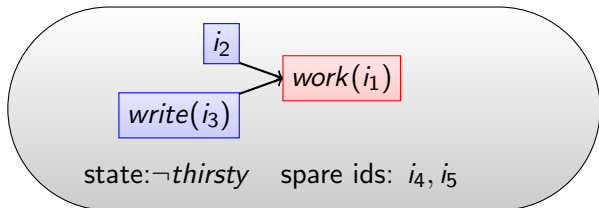
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## Managing precedence constraints



Execute *write*:  $\gamma(\neg thirsty, write(i_3)) = thirsty$

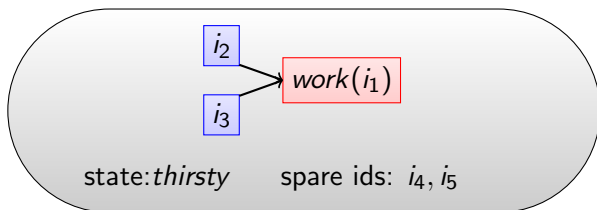
Manage precedence constraints with operators:

- Wait for tasks to be finished
- Free the identifiers
- Mark task as ready to run

Wait op:  $wait_{22}(i_1, i_2, i_3)$

$pre = \{finished(i_2), finished(i_3), \dots\}$   
 $eff = \{free\ i_2\ and\ i_3$   
 $running(i_1)\}$

## Managing precedence constraints



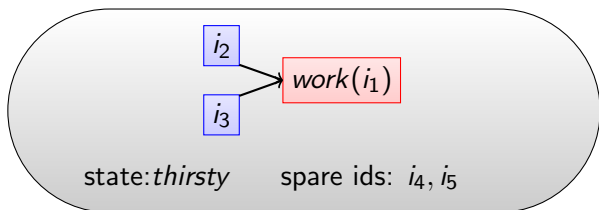
Manage precedence constraints with operators:

- Wait for tasks to be finished
- Free the identifiers
- Mark task as ready to run

Wait op:  $wait_{22}(i_1, i_2, i_3)$

$pre = \{finished(i_2), finished(i_3), \dots\}$   
 $eff = \{free\ i_2\ and\ i_3$   
 $running(i_1)\}$

## Managing precedence constraints



Remove constraints with  $wait_{22}(i_1, i_2, i_3)$

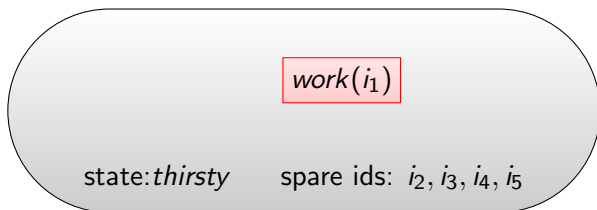
Manage precedence constraints with operators:

- Wait for tasks to be finished
- Free the identifiers
- Mark task as ready to run

Wait op:  $wait_{22}(i_1, i_2, i_3)$

$pre = \{finished(i_2), finished(i_3), \dots\}$   
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 $running(i_1)\}$

## Managing precedence constraints



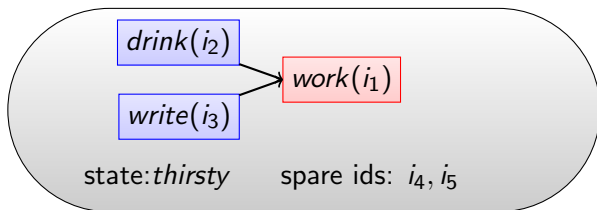
Manage precedence constraints with operators:

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- Free the identifiers
- Mark task as ready to run

Wait op:  $wait_{22}(i_1, i_2, i_3)$

$pre = \{finished(i_2), finished(i_3), \dots\}$   
 $eff = \{free\ i_2\ and\ i_3$   
 $running(i_1)\}$

# Translating Methods: Overview



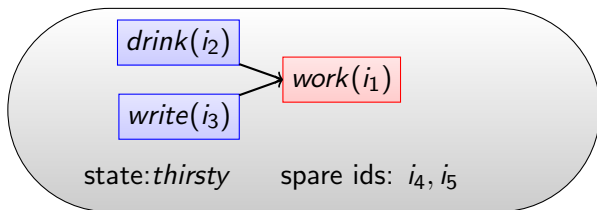
When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

```
Method op:  $work(i_1, i_2, i_3)$   
pre = {  $work(i_1), running(i_1)$  }  
eff = { allocate  $i_1, i_2$   
 $drink(i_2), brew(i_3), work(i_1)$   
 $wait_{22}(i, i_2, i_3),$   
 $running(i_2), running(i_3)$ 
```



# Translating Methods: Overview



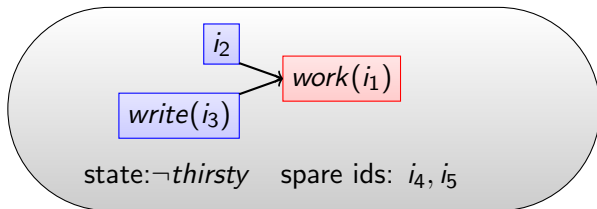
Execute  $drink(i_2)$

Method op:  $work(i_1, i_2, i_3)$   
pre = {  $work(i_1), running(i_1)$  }  
eff = { allocate  $i_1, i_2$   
 $drink(i_2), brew(i_3), work(i_1)$   
 $wait_{22}(i, i_2, i_3),$   
 $running(i_2), running(i_3)$

When translating a method (> 2 tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
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## Translating Methods: Overview

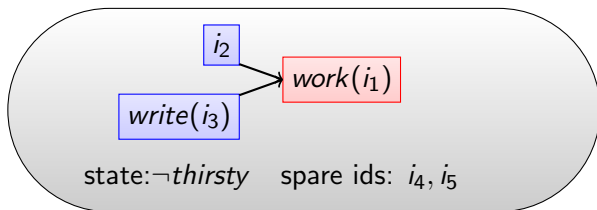


When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

Method op:  $work(i_1, i_2, i_3)$   
pre =  $\{work(i_1), running(i_1)\}$   
eff =  $\{allocate\ i_1, i_2$   
 $drink(i_2), brew(i_3), work(i_1)$   
 $wait_{22}(i, i_2, i_3),$   
 $running(i_2), running(i_3)$

## Translating Methods: Overview



Execute `write(i3)`

Method op: `work(i1, i2, i3)`

`pre = {work(i1), running(i1)}`

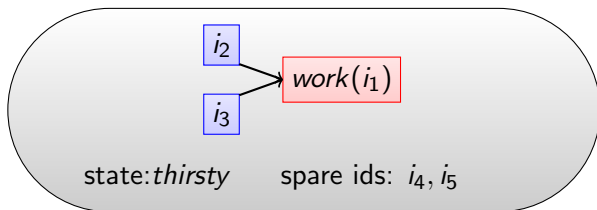
`eff = {allocate i1, i2  
drink(i2), brew(i3), work(i)  
wait22(i, i2, i3),  
running(i2), running(i3)`

`running(i1)`

When translating a method (> 2 tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

## Translating Methods: Overview

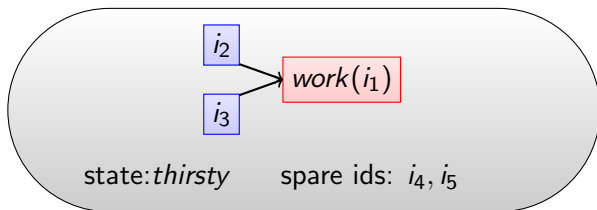


When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

```
Method op: work( $i_1, i_2, i_3$ )  
pre = {work( $i_1$ ), running( $i_1$ )}  
eff = {allocate  $i_1, i_2$   
drink( $i_2$ ), brew( $i_3$ ), work( $i_1$ )  
wait22( $i, i_2, i_3$ ),  
running( $i_2$ ), running( $i_3$ )
```

## Translating Methods: Overview



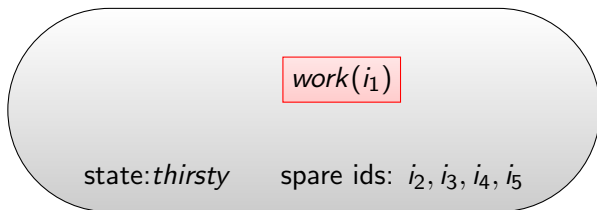
Execute  $wait_{22}$

Method op:  $work(i_1, i_2, i_3)$   
pre = { $work(i_1), running(i_1)$ }  
eff = {allocate  $i_1, i_2$   
 $drink(i_2), brew(i_3), work(i_1)$   
 $wait_{22}(i, i_2, i_3),$   
 $running(i_2), running(i_3)$

When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

## Translating Methods: Overview

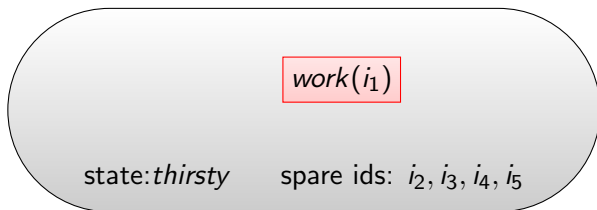


When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

Method op: *work*( $i_1, i_2, i_3$ )  
pre = {*work*( $i_1$ ), *running*( $i_1$ )}  
eff = {allocate  $i_1, i_2$   
*drink*( $i_2$ ), *brew*( $i_3$ ), *work*( $i$ )  
*wait*<sub>22</sub>( $i, i_2, i_3$ ),  
*running*( $i_2$ ), *running*( $i_3$ )

# Translating Methods: Overview



Execute *work*( $i_1, i_2, i_3$ )

Method op: *work*( $i_1, i_2, i_3$ )

pre = {*work*( $i_1$ ), *running*( $i_1$ )}

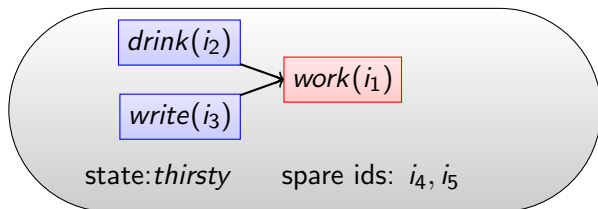
eff = {allocate  $i_1, i_2$   
*drink*( $i_2$ ), *brew*( $i_3$ ), *work*( $i$ )  
*wait*<sub>22</sub>( $i, i_2, i_3$ ),  
*running*( $i_2$ ), *running*( $i_3$ )

.....( $i$ )

When translating a method (> 2 tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

## Translating Methods: Overview



When translating a method ( $> 2$  tasks):

- Allocate identifiers
- Assert task network into state
  - Reuse task identifier for last task
- Mark first tasks as ready to run

```
Method op:  $work(i_1, i_2, i_3)$   
pre = {  $work(i_1), running(i_1)$  }  
eff = { allocate  $i_1, i_2$   
 $drink(i_2), brew(i_3), work(i_1)$   
 $wait_{22}(i, i_2, i_3),$   
 $running(i_2), running(i_3)$ 
```