

# On the Feasibility of Planning Graph Style Heuristics for HTN Planning

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# Hierarchical Task Network (HTN) Planning

- HTN planning is the problem of decomposing an initial task to accomplish into a sequence of executable steps.
- Example applications of HTN planning:
  - Encoding control knowledge for planning
    - Agent planning in robotics and simulation: ARTUE (2010), Johnny (2012)
    - Planning and mission generation for games: KILLZONE 2, Elder Scrolls, Armed Assault)
  - Representing and planning with processes and hierarchies
    - Composing web services (Sirin, 2004; Tang 2013; others)
    - Program configuration (Soltani, 2012)
- More expressive than classical planning
  - Can express undecidable problems
- How does this expressivity affect what HTN heuristics can (and cannot) exist?

# HTN Heuristics (Motivation)

Domain independent heuristics in search:

- Problem spaces in planning are intractably large to search
- A heuristic underestimates the distance to nearest solution, or we pretend it does
- Usually based on solving a relaxed, ground (propositional) version of the problem
  - Relax the semantics of what it means to be a solution
  - Relaxation should take low-order polynomial time to solve
  - Solution to full problem implies solution to relaxed problem (but not vice versa)
- There is currently only one (state-independent) HTN heuristic

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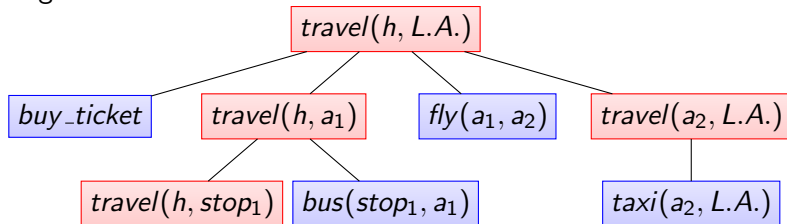
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  - Relaxation should take low-order polynomial time to solve
  - Solution to full problem implies solution to relaxed problem (but not vice versa)
- ~~There is currently only one (state-independent) HTN heuristic~~
- There are only two HTN heuristics
  - Bercher et. al. SoCS 2014

# HTN Planning (Overview)

The purpose of HTN planning is to complete a *task*. Tasks are either

- *Primitive*, which corresponds to some concrete action we know how to perform
  - E.g: *walk(room, hall)*, or *drink(coffee)*
- *Non-primitive*, which is an abstract task. E.g. *travel(home, L.A.)*
- Must recursively decompose non-primitive tasks until we get primitive tasks we know how to execute directly
- We are given a set of *methods*, which are recipes on how to accomplish abstract tasks. E.g., to travel from *home* to *L.A.*, we might



# Propositional HTN Planning: $D = (\mathbf{O}, M)$ , $P = (D, \mathbf{s}_0, tn_0)$

Every HTN problem  $P$  contains an *initial state*  $\mathbf{s}_0$ , which is a collection of propositions  
*Actions* in our domain change the state. Actions have:

- A name (the task)
- A precondition (logical formula over the state)
  - Condition must hold for the action to be executed
- An effect on the state
  - A set of propositions to add
  - A set of propositions to delete

State:

$\{thirsty, tired\}$

Operator: *sleep*

$pre = tired \wedge \neg thirsty$

$eff = \{\neg tired\}$

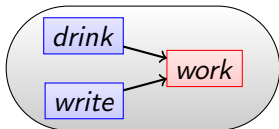
Operator: *drink*

$pre = true$

$eff = \{\neg thirsty\}$

# HTN Planning: $D = (O, M)$ , $P = (D, s_0, \mathbf{tn}_0)$

- An initial *task network*,  $tn_0$ , which is a labeled DAG of *tasks* to accomplish:
  - Edges represent ordering constraints
  - Nodes are labeled with *tasks*, which are either:
    - Primitive tasks (i.e., action names from  $O$ :  $\{drink, write\}$ )
    - Non-primitive tasks (say,  $\{work\}$ )

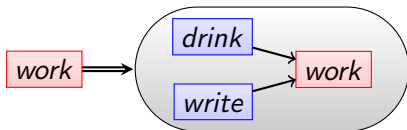


- $M$ , a set of methods to *decompose* non-primitive tasks

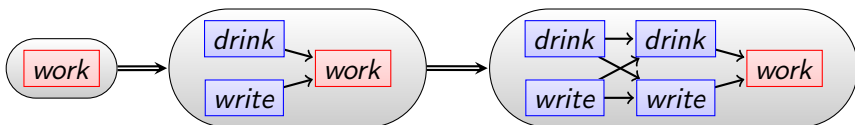
# Methods and Decomposition

- A method  $(t, tn)$  is a non-primitive task  $t$  paired with a network  $tn$

Method:



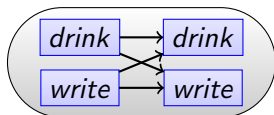
- We *decompose* a task network by replacing a node in the network with a corresponding method's network.





# Primitive Task Networks

- Two states:
  - $thirsty, \neg thirsty$
- Two actions:
  - $\forall_s \gamma(s, write) = thirsty$
  - $\forall_s \gamma(s, drink) = \neg thirsty$



- All names are actions  $\Rightarrow$  network is *primitive*
- It is *executable* in a state if there exists a consistent total order (sequence) which is executable.
- Objective: Decompose initial network to a primitive task network that is executable in  $s_0$ .

# Classical (Propositional STRIPS) Planning

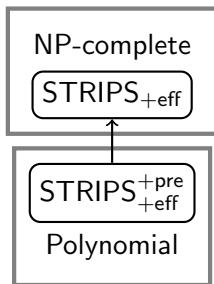
Differences between HTN and classical planning:

- $P = (O, s_0, G)$
- No methods and no initial task network.
- Objective: Find a plan (any executable sequence) that take us to a goal state.
  - No restriction (other than executability) on when planner can insert an action
- PSPACE-complete for propositional problems

# Delete-Free STRIPS

Planning in propositional delete-free STRIPS (Bylander, 1994)

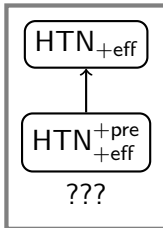
- STRIPS<sub>+eff</sub><sup>+pre</sup> (FF Relaxed GraphPlan)
  - All operators have positive preconditions and effects; positive goal
  - Poly-time to find a plan: just apply operators until a fixed point is reached
  - NP-hard to find optimal plan
  - Basis for many influential classical planning heuristics
- STRIPS<sub>+eff</sub>
  - All operators have positive effects. Arbitrary goal and precondition
  - NP-complete to find a plan (same for optimality)



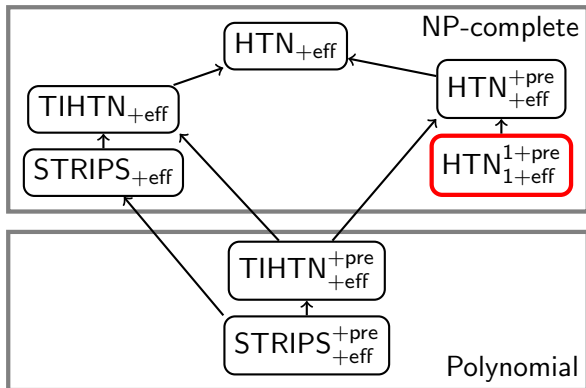
# Delete-Free HTN Planning

Planning in delete-free propositional HTN domains:

- $HTN_{+pre}^{+eff}$ : Positive preconditions and effects
- $HTN_{+eff}$ : Positive effects
- Decidability and complexity previously unknown
- If either of these were in P, we could use them as the base of a heuristic



# HTN<sub>1+eff</sub><sup>1+pre</sup> is NP-hard



Almost all delete-free HTN planning is NP-hard, even the case where:

- Actions have at most 1 positive precondition and effect
- Methods are acyclic, regular (1 non-primitive task), and totally-ordered

# Encoding CNF-SAT in HTN<sub>1+eff</sub><sup>1+pre</sup>

CNF-SAT:  $(V_1 \vee \neg V_3 \vee V_4) \wedge (V_2 \vee V_3 \vee \neg V_4) \wedge (\neg V_1 \vee \neg V_2 \vee \neg V_3)$

For each variable  $V_i$

- Two propositions:  $V_i, \neg V_i$
- Two methods corresponding to truth values of  $V_i$

Method:



Method:



- Method choice determines which of  $V_i, \neg V_i$  is set.
- For last variable, replace last task with  $check_{E_1}$ , instead

$V_1$	$V_2$	$V_3$	$V_4$
$\neg V_1$	$\neg V_2$	$\neg V_3$	$\neg V_4$

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 $\rightarrow$   $nV_1$      $nV_2$      $nV_3$      $nV_4$



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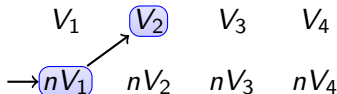
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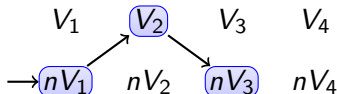
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# Encoding CNF-SAT in HTN<sub>1+pre</sub><sup>1+pre</sup><sub>1+eff</sub>

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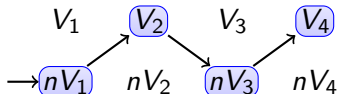
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## Delete-Free HTN planning is NP-hard

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For disjunctive clause  $E_i$ , methods test whether at least one term is true:

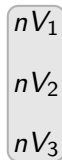
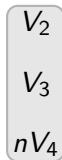
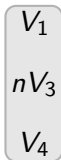
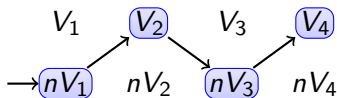
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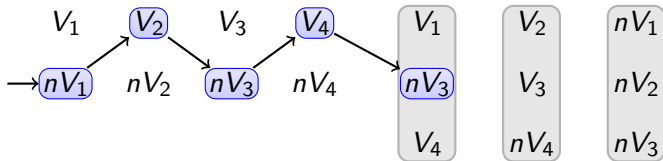
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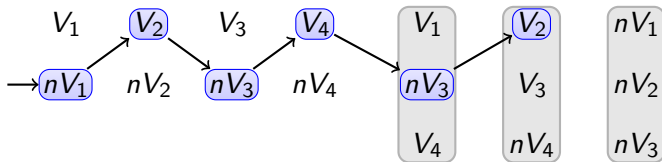
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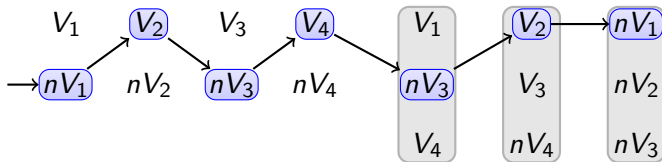
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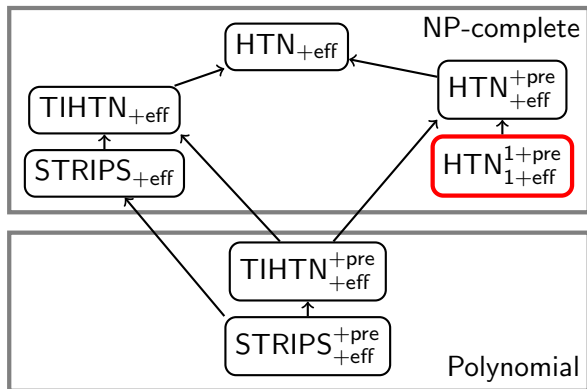
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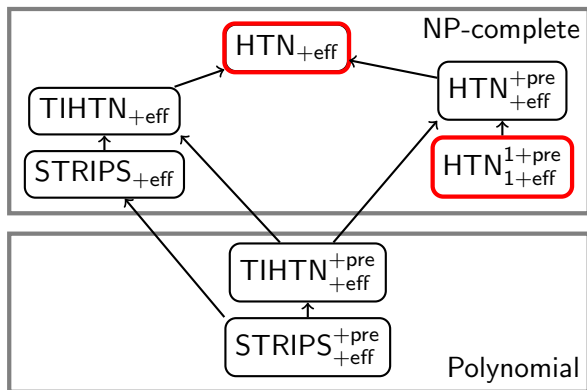
# NP-Hardness and Completeness



- Even highly restricted delete-free HTN planning is NP-hard
- Now we show that every solvable delete-free HTN problem has a polynomial-size witness

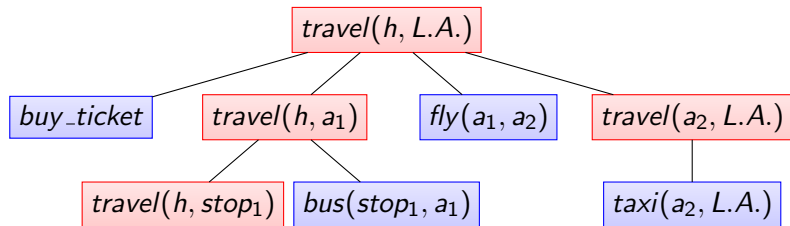


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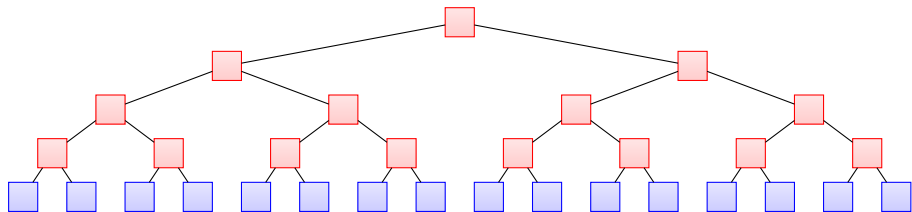
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# HTN Decomposition Trees



- History of decompositions forms a tree where the leaves are the current task network (Geier and Bercher, 2011)
  - If that task network is executable, this tree is a proof that the problem is solvable
- Even if no actions change the state, min tree size may still be exponential
  - To show  $\text{HTN}_{+\text{eff}} \in \text{NP}$ , we need a polynomial size witness

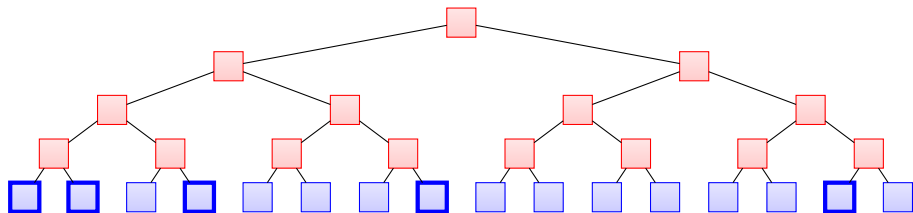
# A Polynomial-Size Witness for $\text{HTN}_{+eff}$



Given a solution:

- In delete-free planning, only as many actions as there are propositions can change the state
- Prune tree to the minimal tree that contains those actions
- Construct a poly-size table to show the pruned tree has an executable expansion
- Use a pumping lemma to shorten the tree to polynomial height
- Poly height, poly width tree and a poly-size table  $\implies \text{HTN}_{+eff} \in \text{NP}$

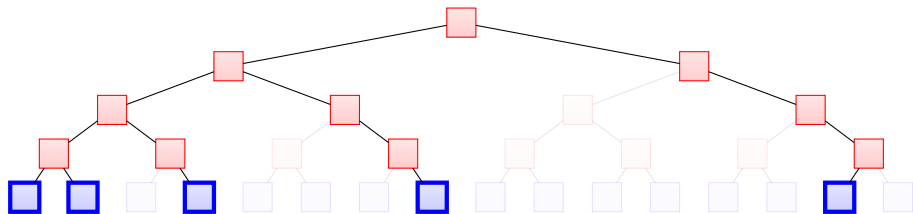
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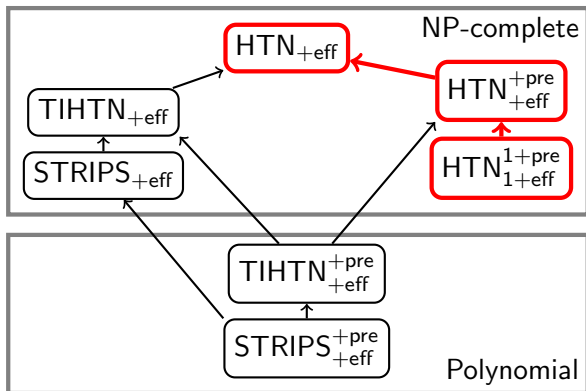
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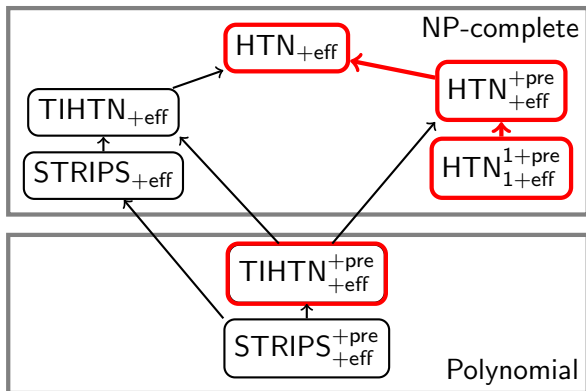
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# Delete-Free HTN planning is NP-complete



- $HTN_{+pre+eff}^{+pre}$  has a poly-size witness and can encode SAT, so it is NP-complete
- Just ignoring the deletes and negative preconditions is not enough to create an HTN heuristic - we must further relax the semantics

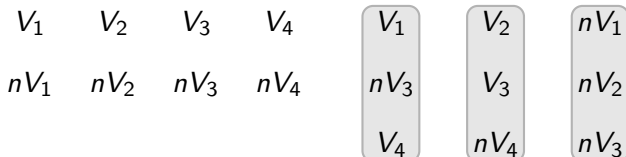
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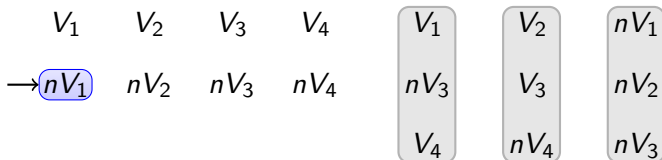
- Task-Insertion HTN (TIHTN) Planning (Geier & Bercher, 2011):
  - Allows insertion of arbitrary operators into task network
- $\text{TIHTN}_{+eff}^{+pre}$  planning is poly-time:
  - Insert operators until you reach a fixpoint
  - Test to see if tasks are executable in the fixpoint state (similar to testing if a CFG is empty).
- Poly-time  $\implies$   $\text{TIHTN}_{+eff}^{+pre}$  is a candidate for an HTN heuristic.
- We can't reduce SAT to  $\text{TIHTN}_{+eff}^{+pre}$ :





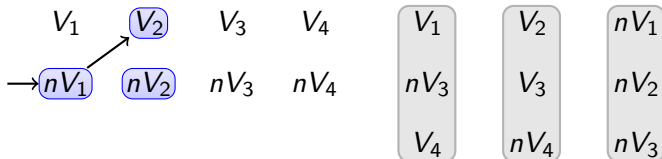
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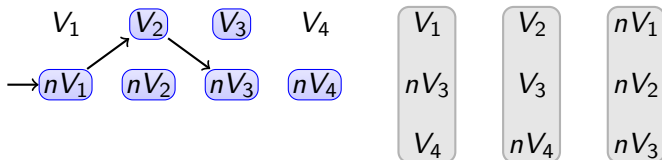
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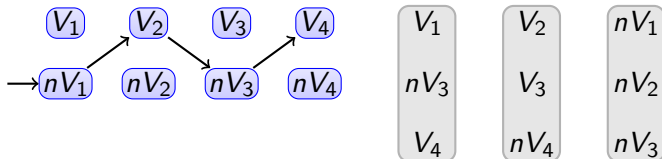
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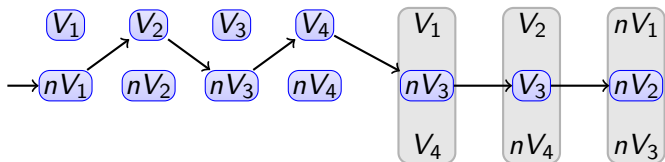
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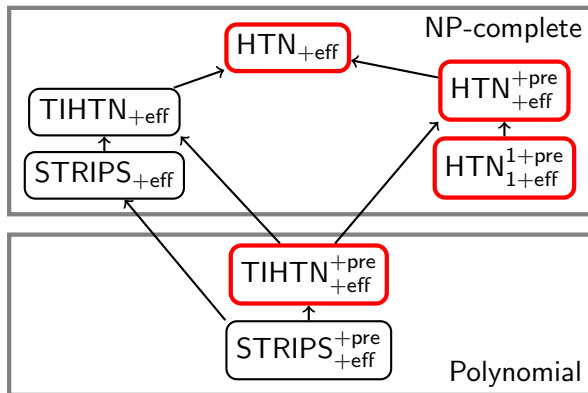


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- Task-Insertion HTN (TIHTN) Planning (Geier & Bercher, 2011):
  - Allows insertion of arbitrary operators into task network
- $\text{TIHTN}_{+eff}^{+pre}$  planning is poly-time:
  - Insert operators until you reach a fixpoint
  - Test to see if tasks are executable in the fixpoint state (similar to testing if a CFG is empty).
- Poly-time  $\implies$   $\text{TIHTN}_{+eff}^{+pre}$  is a candidate for an HTN heuristic.
- We can't reduce SAT to  $\text{TIHTN}_{+eff}^{+pre}$ :



# Conclusions



Delete-free HTN planning is NP-complete!

- Heuristics based on delete-free HTNs must further relax semantics
- $TIHTN_{+pre+eff}^{+pre}$  (HTN planning with task insertion) is one candidate
- Need to explore the trade-offs between complexity & fidelity